DeepLearning on FPGAs

Artifical Neuronal Networks: Image classification

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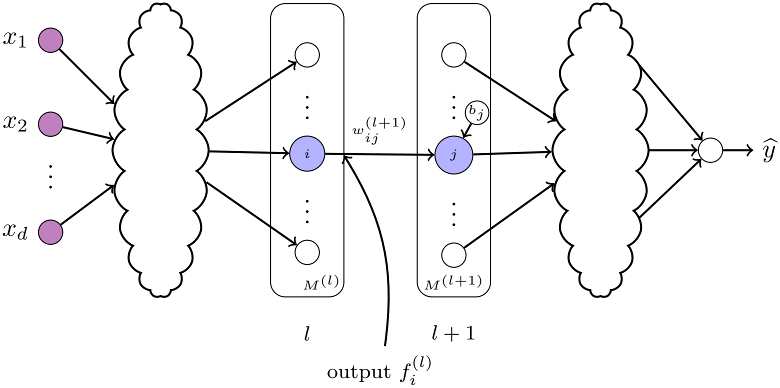
MLPs A more detailed view



$w_{i,j}^{(l+1)} \widehat{=}$ Weight from neuron i in layer l to neuron j in layer $l+1$

$f_j^{(l+1)} = h(\sum_{i=0}^{M^{(l)}} w_{i,j}^{(l+1)} f_i^{(l)} + b_j^{(l+1)})$

Backpropagation for sigmoid activation / RMSE loss

Gradient step:

$$
\begin{aligned}
w_{i,j}^{(l)} &= w_{i,j}^{(l)} - \alpha \cdot \delta_j^{(l)} f_i^{(l-1)} \\
b_j^{(l)} &= b_j^{(l)} - \alpha \cdot \delta_j^{(l)}
\end{aligned}
$$

Recursion:

$$
\begin{aligned}
\delta_j^{(l-1)} &= f_j^{(l-1)} \left(1 - f_j^{(l-1)}\right) \sum_{k=1}^{M^{(l)}} \delta_k^{(l)} w_{j,k}^{(l)} \\
\delta_j^{(L)} &= -\left(y_i - f_j^{(L)}\right) f_j^{(L)} \left(1 - f_j^{(L)}\right)
\end{aligned}
$$

Backpropagation for sigmoid activation / RMSE loss

Gradient step:

$$w_{i,j}^{(l)} = w_{i,j}^{(l)} - \alpha \cdot \delta_j^{(l)} f_i^{(l-1)}$$
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Recursion:

derivative of activation function

$$\delta_j^{(l-1)} = f_j^{(l-1)} \left(1 - f_j^{(l-1)}\right) \sum_{k=1}^{M^{(l)}} \delta_k^{(l)} w_{j,k}^{(l)}$$
$$\delta_j^{(L)} = -\left(y_i - f_j^{(L)}\right) f_j^{(L)} \left(1 - f_j^{(L)}\right)$$

derivative of loss function

Image classification

Our goal: Classify images with Deep learning
Recap: Neuronal Networks need vector input $\vec{x}$

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Question: How are images represented?
Most simple representation: Bitmap of pixels

- Image has fixed number if pixels (height $\times$ width)
- Image has fixed number of color channels (e.g. RGB)
- Every pixel saves the color values of all color channels

Thus: An image is a matrix of pixels with multiple values (=vector) per entry
Sidenote: Mathematically this is called a tensor

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Idea: Map every entry in the pixel matrix to exactly 1 input neuron

Image Representation: Example

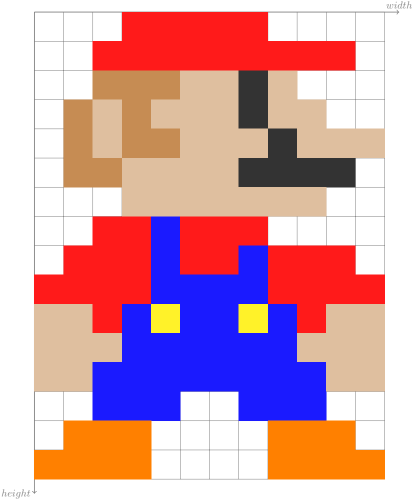
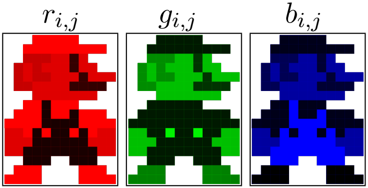


Image: Matrix $M = [\vec{p}_{ij}]_{ij}$
Entry: $\vec{p}_{ij} = (r_{ij}, g_{ij}, b_{ij})^T$



Input neurons:
$\vec{x} = (r_{11}, g_{11}, b_{11}, r_{12}, g_{12}, \dots)^T$

Example: 256×256 RGB image
$\Rightarrow 3 \cdot 256 \cdot 256 = 196.608$ input neurons

Image Representation

Observation 1: Even smaller images need a lot of neurons

- $width \approx 256 - 1920$
- $height \approx 256 - 1080$
- $r_{ij}, g_{ij}, b_{ij} \in \{0, 1, \ldots, 255\}$

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- $height \approx 256 - 1080$
- $r_{ij}, g_{ij}, b_{ij} \in \{0, 1, \ldots, 255\}$

Observation 2: This gets worse, if the neural network is "deep"

- Input-Layer: 196.608 neurons
- First hidden-layer: 1000 neurons
- Second hidden-layer: 100 neurons
- Output layer: 1 neuron

$\Rightarrow 196.608 \cdot 1000 + 1000 \cdot 100 + 100 \cdot 1 = 196.708.100$ weights

Thus: Even for small images we need to learn a lot of weights

Image Representation: Making images smaller

Obviously: Images need to be smaller!

- Merge a $r \times r$ grid of pixels into a single pixel by applying reduction kernel channel-wise $k_c : \mathbb{N}^r \to \mathbb{N}$ over all pixels
- By defining appropriate kernels, we can achieve smoothing, anti-alising etc.

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Note: Pixel values are integers (e.g. $0 - 255$). Reduction kernels can be defined over $\mathbb{R}$, meaning $k_c : \mathbb{R}^r \rightarrow \mathbb{R}$. Then values need to be mapped to integers again:

$$\tilde{k}_c = max(0, min(255, \lfloor k_c \rfloor))$$

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$$\tilde{k}_c = max(0, min(255, \lfloor k_c \rfloor))$$

Thus: Assume appropriate mapping and use $k_c : \mathbb{R}^r \rightarrow \mathbb{R}$

Reduction kernel: Example

Simple and fast: Averaging $k_c = \frac{1}{r} \sum_{i=1}^{r} c_i$

160	210	133	111
88	39	70	130
110	240	10	120
100	66	88	93

Padding: The way you handle unknown inputs (e.g. image-border)
Overlapping: The way you move the grid over the image
Here: Kernel is applied non-overlapping with no padding

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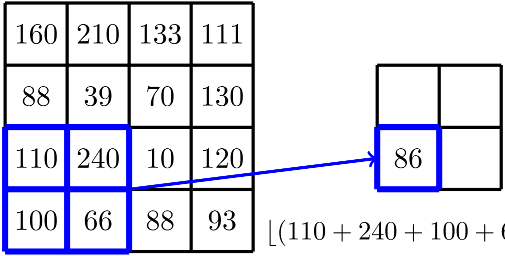

$\lfloor(110 + 240 + 100 + 66) \cdot 0.25\rfloor = 86$

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160	210	133	111
88	39	70	130
110	240	10	120
100	66	88	93

86	120

$$\lfloor (10 + 120 + 88 + 93) \cdot 0.25 \rfloor = 120$$

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160	210	133	111
88	39	70	130
110	240	10	120
100	66	88	93

81	
86	120

$\lfloor (160 + 210 + 88 + 39) \cdot 0.25 \rfloor = 81$

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Reduction kernel: Example

Simple and fast: Averaging $k_c = \frac{1}{r}\sum_{i=1}^{r} c_i$

160	210	133	111
88	39	70	130
110	240	10	120
100	66	88	93

81	153
86	120

$\lfloor (133 + 111 + 70 + 130) \cdot 0.25 \rfloor = 153$

Padding: The way you handle unknown inputs (e.g. image-border)
Overlapping: The way you move the grid over the image
Here: Kernel is applied non-overlapping with no padding

Image Representation: Making images smaller (2)

Observation 1: We can apply the same kernel in many different ways → Pixel-padding and/or overlapping might occur[1]

For now: We assume non-overlapping application with no padding
But: Other application schemes can obviously be implemented

[1] Animations see: https://github.com/vdumoulin/conv_arithmetic

Image Representation: Making images smaller (3)

Observation 2: The average kernel uses the same coefficient $\frac{1}{r}$

$$k_c = \frac{1}{r}\sum_{i=1}^{r} c_i = \sum_{i=1}^{r} \frac{1}{r} \cdot c_i$$

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$$k_c = \frac{1}{r}\sum_{i=1}^{r} c_i = \sum_{i=1}^{r} \frac{1}{r} \cdot c_i$$

More general: Convolution using arbitrary weights w_i

$$k_c = \sum_{i=1}^{r} w_i \cdot c_i = \vec{w} * \vec{c}$$

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$$k_c = \sum_{i=1}^{r} w_i \cdot c_i = \vec{w} * \vec{c}$$

Note: This is basically a weighted sum!
But name-overloading here: Convolution is a well-known operation in signal processing and statistics

Convolution: Some intuitions

In system theory: Given a system with a transfer-function f we can compute its reaction to an input signal g by computing the convolution $f * g = \int f(\tau)g(t-\tau)d\tau$

In statistics: Given two time series as continuous functions f and g, we can measure the similarity of these two functions by computing the cross-correlation $f \star g = \int f(\tau)g(t+\tau)d\tau$

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Note: Both are basically the same with different perspective and a slightly different index-shift
Bottom-Line: A kernel reacts to specific parts of a function / signal / image, thus **filtering** out important features
$\Rightarrow$ This is some kind of feature extraction

Convolution: Example

Note: In discrete convolution integrals become summation:

$$k_c = \sum_{i=1}^{r} w_i \cdot c_i = \vec{w} * \vec{c}$$

170	20	153	11
122	39	70	200
180	80	10	120
20	120	45	140

image

$*$

1	−0.5
−0.5	1

kernel / weights / filter

$=$

result

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image:

170	20	153	11
122	39	70	200
180	80	10	120
20	120	45	140

$*$

kernel / weights / filter:

1	-0.5
-0.5	1

$=$

result:

250	

$$180 \cdot 1 - 80 \cdot 0.5 - 20 \cdot 0.5 + 120 \cdot 1 = 250$$

Convolution: Example

Note: In discrete convolution integrals become summation:

$$k_c = \sum_{i=1}^{r} w_i \cdot c_i = \vec{w} * \vec{c}$$

image:

170	20	153	11
122	39	70	200
180	80	10	120
20	120	45	140

$*$

kernel / weights / filter:

1	−0.5
−0.5	1

$=$

result:

250	67

$$10 \cdot 1 - 120 \cdot 0.5 - 45 \cdot 0.5 + 140 \cdot 1 = 67$$

image   kernel / weights / filter   result

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$$k_c = \sum_{i=1}^{r} w_i \cdot c_i = \vec{w} * \vec{c}$$

170	20	153	11
122	39	70	200
180	80	10	120
20	120	45	140

image

$*$

1	−0.5
−0.5	1

kernel / weights / filter

$=$

138	
250	67

result

$$170 \cdot 1 - 20 \cdot 0.5 - 122 \cdot 0.5 + 39 \cdot 1 = 138$$

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170	20	153	11
122	39	70	200
180	80	10	120
20	120	45	140

image

$*$

1	−0.5
−0.5	1

kernel / weights / filter

$=$

138	255
250	67

result

$$153 \cdot 1 - 11 \cdot 0.5 - 70 \cdot 0.5 + 200 \cdot 1 = 255$$

Convolutional neural networks (CNN)

Observation 1: Convolution can reduce the size of images
Observation 2: Convolution can perform feature extraction
Observation 3: Neural networks can learn weights $\vec{w}$
$\Rightarrow$ Convolutional neural networks (CNN) ($\sim$ **LeCun, 1989**)

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Idea: Every convolutional layer has its own weight matrix

- Move convolution kernel over input data (with padding etc.)
- Apply activation function to create another (smaller) image
- Once the images are small enough, use fully connected layers
- During backpropagation, compute errors for the kernel weights

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Question: How do we compute the kernel weights?
Short: Use backpropagation - **Long:** We need some more notation

CNNs: Some remarks

Note 1: Since convolution is used internally, there is no need for mapping values **inside** the net $\rightarrow$ use computed values directly

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Note 5: CNNs somewhat model receptive fields in biology

CNN: Notation and weight sharing

f_{00}	f_{01}	f_{02}
f_{10}	f_{11}	f_{12}
f_{20}	f_{21}	f_{22}

input $\vec{f}$

$*$

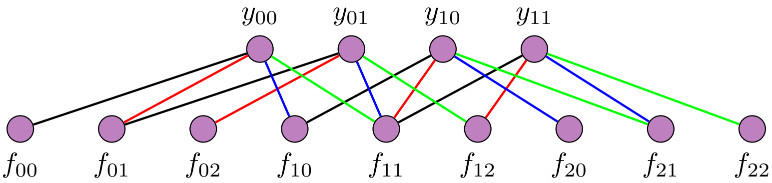
w_{00}	w_{01}
w_{10}	w_{11}

weights $\vec{w}$

$=$

$w_{00}f_{00} + w_{01}f_{01} + w_{10}f_{10} + w_{11}f_{11}$	$w_{00}f_{01} + w_{01}f_{02} + w_{10}f_{11} + w_{11}f_{12}$
$w_{00}f_{10} + w_{01}f_{11} + w_{10}f_{20} + w_{11}f_{21}$	$w_{00}f_{11} + w_{01}f_{12} + w_{10}f_{21} + w_{11}f_{22}$

output $\vec{y}$



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f_{00}	f_{01}	f_{02}
f_{10}	f_{11}	f_{12}
f_{20}	f_{21}	f_{22}

$*$

Weights $\vec{w}$:

w_{00}	w_{01}
w_{10}	w_{11}

$=$

Output $\vec{y}$:

$w_{00}f_{00} + w_{01}f_{01} + w_{10}f_{10} + w_{11}f_{11}$	$w_{00}f_{01} + w_{01}f_{02} + w_{10}f_{11} + w_{11}f_{12}$
$w_{00}f_{10} + w_{01}f_{11} + w_{10}f_{20} + w_{11}f_{21}$	$w_{00}f_{11} + w_{01}f_{12} + w_{10}f_{21} + w_{11}f_{22}$

input $\vec{f}$ weights $\vec{w}$ output $\vec{y}$

Mathematically (here with cross-correlation):

$$y_{i,j}^{(l)} = \sum_{i'=0}^{M^{(l)}} \sum_{j'=0}^{M^{(l)}} w_{i,j}^{(l)} \cdot f_{i+i',j+j'}^{(l-1)} + b_{i,j}^{(l)} = w^{(l)} * f^{(l-1)} + b^{(l)}$$

$$f_{i,j}^{(l)} = h(y_{i,j}^{(l)})$$

$M^{(l)} \times M^{(l)}$ bias **matrix**!

CNN: How to compute $\frac{\partial E}{\partial w_{i,j}^{(l)}}$ and $\frac{\partial E}{\partial b_{i,j}^{(l)}}$?

f_{00}	f_{01}	f_{02}
f_{10}	f_{11}	f_{12}
f_{20}	f_{21}	f_{22}

input $\vec{f}$

$*$

w_{00}	w_{01}
w_{10}	w_{11}

weights $\vec{w}$

$=$

$w_{00}f_{00} + w_{01}f_{01} + w_{10}f_{10} + w_{11}f_{11}$	$w_{00}f_{01} + w_{01}f_{02} + w_{10}f_{11} + w_{11}f_{12}$
$w_{00}f_{10} + w_{01}f_{11} + w_{10}f_{20} + w_{11}f_{21}$	$w_{00}f_{11} + w_{01}f_{12} + w_{10}f_{21} + w_{11}f_{22}$

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...

...

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Backpropagation for sigmoid activation

Gradient step:

$$
\begin{aligned}
w_{i,j}^{(l)} &= w_{i,j}^{(l)} - \alpha \cdot \delta^{(l)} * rot180(f)^{(l-1)} f_{i,j}^{(l-1)} \\
b_j^{(l)} &= b_j^{(l)} - \alpha \cdot \delta_j^{(l)}
\end{aligned}
$$

Recursion:

$$\delta^{(l+1)} = \delta^{(l)} * rot180(w^{(l+1)}) \cdot f_{i,j}^{(l)}(1 - f_{i,j})^{l}$$

Backpropagation for sigmoid activation

Gradient step:

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Recursion:

$$
\delta^{(l+1)} = \delta^{(l)} * rot180(w^{(l+1)}) \cdot f_{i,j}^{(l)}(1 - f_{i,j})^l
$$

$$
rot180 \begin{array}{|c|c|} \hline w_{10} & w_{11} \\ \hline w_{00} & w_{01} \\ \hline \end{array} = \begin{array}{|c|c|} \hline w_{01} & w_{00} \\ \hline w_{11} & w_{10} \\ \hline \end{array}
$$

Backpropagation for activation h

Gradient step:

$$\begin{aligned} w_{i,j}^{(l)} &= w_{i,j}^{(l)} - \alpha \cdot \delta^{(l)} * rot180(f)^{(l-1)} f_{i,j}^{(l-1)} \\ b_j^{(l)} &= b_j^{(l)} - \alpha \cdot \delta_j^{(l)} \end{aligned}$$

Recursion:

$$\delta^{(l+1)} = \delta^{(l)} * rot180(w^{(l+1)}) \cdot \frac{\partial h(y_i^{(l)})}{\partial y_i^{(l)}}$$

Observation: A convolution during forward-step results in cross-correlation on the backward step and vice-versa
Note: The values (and thus positions) of the weights are learnt
Thus: Does not matter if we implement convolution or corss-correlation. Just need to "reverse" it during backprop.

CNN: Some architectural remarks

So far: We assumed 1 color channel - what about 3 channels?

Idea 1: Merge color channels into single value

- **Average:** $(r_{i,j} + g_{i,j} + b_{i,j})\,/3$
- **Lightness:** $(max\,(r_{i,j}, g_{i,j}, b_{i,j}) - min\,(r_{i,j}, g_{i,j}, b_{i,j}))\,/2$
- **Luminosity:** $0.21 r_{i,j} + 0.72 g_{i,j} + 0.07 r_{i,j}$

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- **Lightness:** $(max(r_{i,j}, g_{i,j}, b_{i,j}) - min(r_{i,j}, g_{i,j}, b_{i,j}))/2$
- **Luminosity:** $0.21r_{i,j} + 0.72g_{i,j} + 0.07r_{i,j}$

Observation: Average and Luminosity look like weighted sums...

$\rightarrow$ Given $k^{(l)}$ input channels in layer l, for every pixel j do:

$$f_j^{(l)} = h\left(\sum_{k=1}^{k^{(l)}} f_j^{(l-1)} \cdot w_{k,j}^{(l)} + b_j\right)$$

Thus: Use standard backprop. to learn weights

CNN: Some architectural remarks (2)

Idea 2: Use 1 weight matrix per channel and extract 1 feature
More general: Perform $k^{(l)}$ convolutions per layer

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- Use and learn $k^{(l)}$ weight matrices per layer
- Generating $k^{(l)}$ smaller images per layer
- So that multiple features are extracted per layer

$\Rightarrow$ Build a tree-like convolution structure, where more sophisticated features are extracted based on already extracted features

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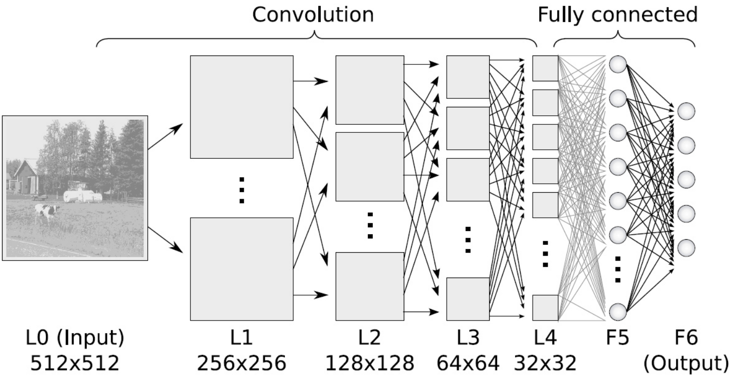
More general: Perform $k^{(l)}$ convolutions per layer

- Use and learn $k^{(l)}$ weight matrices per layer
- Generating $k^{(l)}$ smaller images per layer
- So that multiple features are extracted per layer

$\Rightarrow$ Build a tree-like convolution structure, where more sophisticated features are extracted based on already extracted features
Finally: Use fully connected layers to perform classification

Usually: A combination is used between feature extraction and channel reduction

CNN: Example[2]



[2]Source: http://www.ais.uni-bonn.de/deep_learning/images/Convolutional_NN.jpg

CNN: Some architectural remarks (3)

Idea 2 With color channels

$$
\begin{aligned}
y_{i,j}^{(l)} &= \sum_{c=1}^{3} \sum_{i'=0}^{M^{(l)}} \sum_{j'=0}^{M^{(l)}} w_{i,j,c}^{(l)} \cdot f_{i+i',j+j',c}^{(l-1)} + b_{i,j,c}^{(l)} \\
f_{i,j}^{(l)} &= h(y_{i,j}^{(l)})
\end{aligned}
$$

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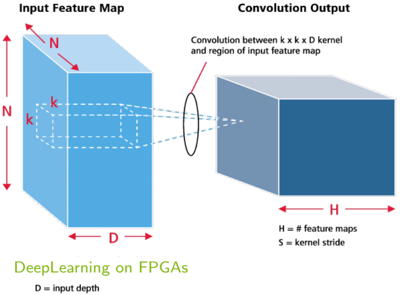
\begin{aligned}
y_{i,j}^{(l)} &= \sum_{c=1}^{3} \sum_{i'=0}^{M^{(l)}} \sum_{j'=0}^{M^{(l)}} w_{i,j,c}^{(l)} \cdot f_{i+i',j+j',c}^{(l-1)} + b_{i,j,c}^{(l)} \\
f_{i,j}^{(l)} &= h(y_{i,j}^{(l)})
\end{aligned}
$$



Thus
Basically the same, but one additional dimension

CNN: Some architectural remarks (4)

Sometimes: We want to reduce the image size even further without too much computation
Downsampling/Pooling: Merge a $r \times r$ grid into a single pixel

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- **Max:** $f_{i,j}^{(l)} = max\left(p_{i,j}, p_{i,j+1}, \ldots p_{i+r,j+r}\right)$
- **Avg:** $f_{i,j}^{(l)} = \frac{1}{r \cdot r} \sum_{i'=0}^{r} \sum_{j'=0}^{r} p_{i+i',j+j'}$
- **Sum:** $f_{i,j}^{(l)} = \sum_{i'=0}^{r} \sum_{j'=0}^{r} p_{i+i',j+j'}$

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Note: This is the same as convolution, but without parameters

Thus: No backpropagation-step needed for this layer

$\Rightarrow$ Just "upsample" delta-values from next layer and backward upsampled values to the previous layer

CNN: Some implementation remarks

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$\rightarrow$ implementation should be freely combinable with activation function and other layers

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But: Size of output also depends on padding / striding approach
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$\rightarrow$ For compilers layer-sizes should be known at compile time
$\Rightarrow$ Define a compile-time macro / template for easier programming, but high speed implementation

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⇒ Define a compile-time macro / template for easier programming, but high speed implementation

Obviously 2: Pooling is a special kind of layer
Note: Backprop. is not required here, but just correct sampling

CNN: Some implementation remarks (2)

Parallelism: Neural network offer three kind of parallism

First: On feature-extraction level

→ We can perform every convolution per layer in full parallel

Note: This requires some form of synchronization once we reach the fully-connected layer

CNN: Some implementation remarks (2)

Parallelism: Neural network offer three kind of parallism

First: On feature-extraction level

→ We can perform every convolution per layer in full parallel

Note: This requires some form of synchronization once we reach the fully-connected layer

Second: On computational level

→ A convolution requires $r \times r$ independent multiplications

$$\sum_{i'=0}^{M^{(l)}} \sum_{j'=0}^{M^{(l)}} w_{i,j}^{(l)} \cdot f_{i+i',j+j'}^{(l-1)} + b_{i,j}^{(l)} = w^{(l)} * f^{(l-1)} + b^{(l)}$$

Additionally: Activation function needs to be evaluated independently for every pixel

CNN: Some implementation remarks (3)

Third: On gradient level
$\rightarrow$ Perform gradient computations in parallel on parts of the data
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CNN: Some implementation remarks (3)

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$\rightarrow$ Perform gradient computations in parallel on parts of the data
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Note:

1) is always possible for convolutional networks
2) is usally done by the compiler, if the system supports vectorization instructions (More later)
3) is always possible and the go-to method

CNN: Network architecture

Question: So whats a good network architecture?
Answer: As always, depends on the problem. But the same general ideas as with MLPs still hold.

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Answer: As always, depends on the problem. But the same general ideas as with MLPs still hold.
Additionally for image classification:

- Grayscaled images usually give already a fair performance
- Input images should have the same dimension
- Convolution kernels should be large enough to capture features, but small enough to be fast to compute. Usually we use $1 \times 1 - 7 \times 7$
- Convolution tends to overfit, so regularization should be used
- Deeper architectures usually perform well with pooling

Summary

Important concepts:

- **Convolution** is an important concept in image classification
  - We can extract image features on every part of the image
  - We share parameters in small kernel matrices
- **For image classification** we combine convolution layers and fully-connected layers with backpropagation
- **Sometimes** pooling is necessary

Homework until next meeting

- Extend your backpropagation implementation to a more general approach $\rightarrow$ variable number of neurons etc.
- Design a fully connected neurnal network for MNIST

Whats your accuracy?