DeepLearning on FPGAs

Introduction to Deep Learning

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October 21, 2017

Recap Computer Science Approach

Technical Problem	Classification ✓
Mathematical Method	Perceptron ✓
Algorithm	Simple learning rule ✓
Implementation	System and language ✓

Recap Data Mining

Important concepts:

- **Classification** is one data mining task
- **Training data** is used to define and solve the task
- **A Method** is a general approach / idea to solve a task
- **A algorithm** is a way to realise a method
- **A model** forms the extracted knowledge from data
- **Accuracy** measures the model quality given the data

Recap Perceptron classifier

A perceptron is a linear classifier $f\colon \mathbb{R}^d \to \{0,1\}$ with

$$\widehat{f}(\vec{x}) = \begin{cases} +1 & \text{if } \sum_{i=1}^{d} w_i \cdot x_i \geq b \\ 0 & \text{else} \end{cases}$$

For learning

$\vec{w} = rand(1, \ldots, d+1)$
while ERROR **do**
    for $(\vec{x}_i, y_i) \in \mathcal{D}$ **do**
        $\vec{w} = \vec{w} + \alpha \cdot \vec{x}_i \cdot (y_i - \widehat{f}(\vec{x}_i))$
    end for
end while

Homework

So Who did the homework?

Homework

So Who did the homework?

And How good was your prediction?

Homework

So Who did the homework?
And How good was your prediction?

Some of my results

- 0 vs 1: 99.9% accuracy
- 1 vs 2: 98.6% accuracy
- 3 vs 6: 98.8% accuracy
- 5 vs 6: 94.6% accuracy
- 8 vs 9: 97.4% accuracy

Runtime $\sim 3s$ per model with 100 runs over data
Machine Laptop with Intel i7-4600U @ 2.10GHz, 8GB RAM
Tip Compile with `-O3 -march -mnative`

Data Mining Features are important

Fact 1 State space grows exponentially with increasing dimension.
Example $\mathcal{X} = \{1, 2, \ldots, 10\}$

For $\mathcal{X}^1$, there are 10 different observations
For $\mathcal{X}^2$, there are $10^2 = 100$ different observations
For $\mathcal{X}^3$, there are $10^3 = 1000$ different observations ...

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We usually have no influence on the type of training data
We usually cannot interfere with the real-world process

Thus Training data should be considered incomplete and noisy

Data Mining Features are important (2)

Wolpert 1996 There is no free lunch
Every method has is advantages and disadvantages
Most methods are able to perfectly learn a given toy data set
Problem occurs with noise, outlier and generalisation

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But Some methods prefer certain representations

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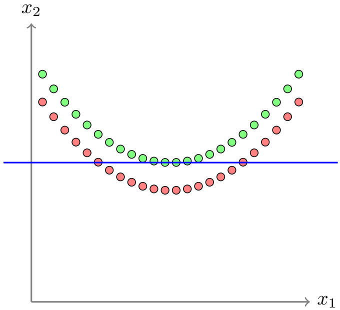
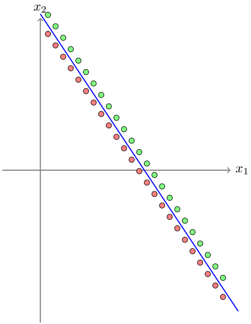
Feature Engineering Finding the right representation for data
Reduce dimension? Increase dimension?
Add additional information? Regularities?
Transform data completely?

Data Mining Features are important (3)



$\xrightarrow{\phi}$



Raw data without transformation.
Linear model is a bad choice.
Parabolic model would be better.

Data transformed with
$\phi(x_1, x_2) = (x_1, x_2 - 0.3 \cdot x_1^2)$.
Now linear model fits the problem.

Data Mining Features are important (4)

Conclusion: Good features are crucial for good results!
Question: How to get good features?

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1. **By hand:** Domain experts and data miner examine the data and try different features based on common knowledge.
2. **Semi supervised:** Data miner examines the data and tries different similarity functions and classes of methods
3. **Unsupervised:** Data miner only encodes some assumptions about regularities into the method.

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Note 1: Hand-crafted features give us insight about the process
Note 2: Semi/unsupervised features give us insight about the data
Our focus: Unsupervised feature extraction.

Our Goal End-to-End learning

Our focus Unsupervised feature extraction
$\rightarrow$ "End-To-End learning"

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So far
Deep Learning seems to be the best method

So...
What is Deep Learning?

Deep Learning Basics

Well... its currently one of the big things in AI!

- **Since 2010:** DeepMind learns and plays old Atari games
- **Since 2012:** Google is able to find cats in youtube videos
- **December 2014:** Near real-time translation in Skype
- **October 2015:** AlphaGo beats the European Go champion
- **October 2015:** Tesla deploys Autopilot in their cars
- **March 2016:** AlphaGo beats the Go Worldchampion
- **June 2016:** Facebook introduces DeepText
- **August 2017:** Facebook uses neural-based translation
- ...

Deep Learning Example

Deep Learning Basics

Deep Learning is a branch of Machine Learning dealing with

- (Deep) Artificial Neural Networks (ANN)
- High Level Feature Processing
- Fast Implementations

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ANNs are well known! So what's new about it?

- We have more data and more computation power
- We have a better understanding of optimization
- We use a more engineering-style approach

Our focus now Artificial Neural Networks

Data Mining Model optimization

Important We need some basics about optimization

Recap

$$\vec{w} = \vec{w} + \alpha \cdot \vec{x}_i \cdot (y_i - \widehat{f}(\vec{x}_i))$$

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So far We formulated an **optimization** algorithm to find perceptron weights that minimize classification **error**

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This is a common approach in Data Mining:

- Specify model family
- Specify optimization procedure
- Specify a cost / loss function

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Note: Loss function $\neq$ Accuracy
$\rightarrow$ The loss function is minimized during learning
$\rightarrow$ Accuracy is used to measure the model's quality after learning

Data Mining Stochastic gradient descent (SGD)

Given

A loss function E, the model parameter $\vec{\theta}$, learning rate α_t

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Framework

$\vec{\theta} = random()$

while ERROR **do**

  choose random $(\vec{x}, y) \in \mathcal{D}$

  $\vec{\theta} = \vec{\theta} - \alpha_t \cdot \frac{\partial E(x,y)}{\partial \vec{\theta}}$

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e.g. 100 iterations
e.g. minimum change in θ

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e.g. 100 iterations
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(estimated) gradient of loss depends on θ and (x, y)

Data Mining Perceptron Learning

Observation We implicitly did this for the perceptron

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So The perceptron works well and follows a general framework

Data Mining The XOR Problem

Question What happens if data is not linear separable?

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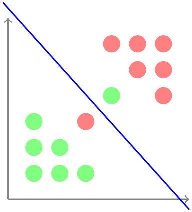
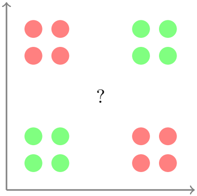


Data linear separable, but noisy



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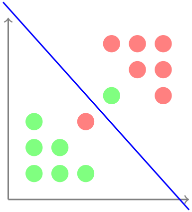

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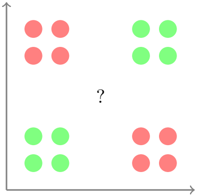


Data not linear separable

Answer Algorithm will never converge, thus

- Use fixed number of iterations
- Introduce some acceptable error margin

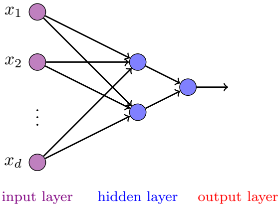
Data Mining Idea - use more perceptrons

Recap (Hand crafted) Feature transformation always possible

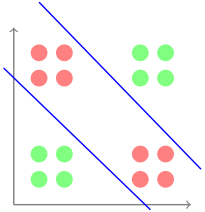
But What about an automatic way?

Rosenblatt 1961

Use multiple perceptrons $\rightarrow$ Multi-Layer Perceptron (MLP)



Biological view



Geometrical view

Data Mining MLP learning

Goal We need to learn weights w / bias b for each perceptron
So far We intuitively derived a learning algorithm

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So far We intuitively derived a learning algorithm
Now Follow stochastic gradient descent algorithm
Loss function (MSE)

$$\ell(\mathcal{D}, \widehat{w}) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\Big(y_i - \widehat{f}(\vec{x}_i)\Big)^2}$$

Observation We need to take the derivative of the loss function

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Observation We need to take the derivative of the loss function
But Loss functions looks complicated
Observation 1 Square-Root is monotone
Observation 2 Constant factor does not change optimization

Data Mining MLP learning (2)

New loss function

$$
\begin{aligned}
\ell(\mathcal{D}, \widehat{w}) &= \frac{1}{2}\left(y_i - \widehat{f}(\vec{x}_i)\right)^2 \\
\nabla_{\widehat{w}} \ell(\mathcal{D}, \widehat{w}) &= \frac{1}{2} 2(y_i - \widehat{f}(\vec{x}_i)) \frac{\partial \widehat{f}(\vec{x}_i)}{\partial \widehat{w}}
\end{aligned}
$$

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$$\widehat{f}(\vec{x}) = \begin{cases} +1 & \text{if } \sum_{i=1}^{d} w_i \cdot x_i + b \geq 0 \\ 0 & \text{else} \end{cases}$$

Observation f is not continuous in 0 (it makes a step)
Thus Impossible to derive $\nabla_{\widehat{w}}\ell(\mathcal{D}, w)$ in 0, because f is not differentiable in 0!

Data Mining MLP activation function

Another problem Combinations of linear functions are still linear

$$
\begin{aligned}
f(x) &= 5x + 3 \\
g(x) &= 10x_1 - 5x_2 \\
f(g(x)) &= 5(10x_1 - 5x_2) + 3 = 50x_1 - 25x_2 + 3
\end{aligned}
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Solution

We need to make f continuous

We need to introduce some non-linearity

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Observation

The input of a perceptron depends on the output of previous one

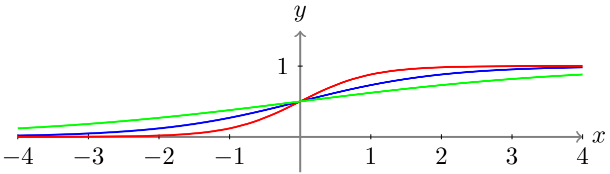
Thus

Apply non-linear **activation** function to perceptron output

Data Mining MLP activation function (2)

Bonus This seems to be a little closer to real neurons
Constraint Activation should be easy to compute

Idea Use sigmoid function



$$\sigma(z) = \frac{1}{1 + e^{-\beta \cdot z}}, \beta \in \mathbb{R}_{>0}$$

Note β controls slope around 0

Data Mining Sigmoid derivative

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&= \beta(1-\sigma(z))\sigma(z)
\end{aligned}
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For inference We compute $\sigma(z)$
For training We compute $\beta\sigma(z)(1-\sigma(z))$
Thus Store activation $\sigma(z)$ for fast computation

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Thus Output perceptron also needs sigmoid activation
But For different labels (e.g. $\{-1, +1\}$) use another activation

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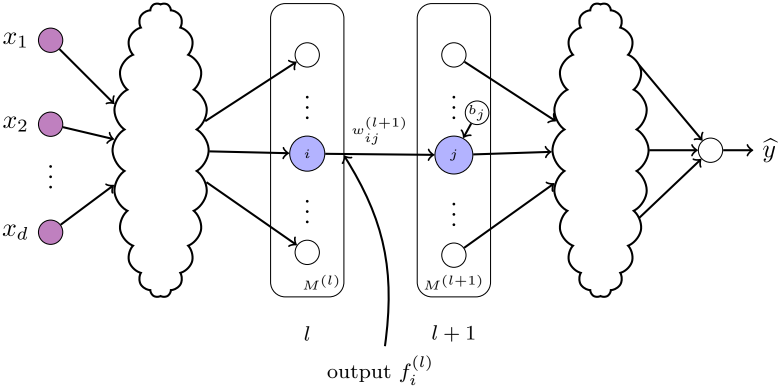
Still
We need to compute $\frac{\partial E(x,y)}{\partial w_{i,j}^{(l)}}$ and $\frac{\partial E(x,y)}{\partial b_j^{(l)}}$
Thus We need a more notation

MLPs A more detailed view



$w_{i,j}^{(l+1)} \widehat{=}$ Weight from neuron i in layer l to neuron j in layer $l+1$

$f_j^{(l+1)} = h(\sum_{i=0}^{M^{(l)}} w_{i,j}^{(l+1)} f_i^{(l)} + b_j^{(l+1)})$

Towards learning MLPs

Goal

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Recap Chain-Rule

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Recap Chain-Rule

$$\frac{\partial}{\partial x}(3x+5)^2 = 2 \cdot (3x+5) \cdot 3 = 6(3x+5)$$

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Recap Chain-Rule

$$\frac{\partial}{\partial x}(3x+5)^2 = 2 \cdot (3x+5) \cdot 3 = 6(3x+5)$$

More formally

Given two functions $f\colon \mathbb{R}^m \to \mathbb{R}$ and $g\colon \mathbb{R}^k \to \mathbb{R}^m$. Let $\vec{u} = g(\vec{x})$ and $\vec{x} \in \mathbb{R}^k$:

$$\frac{\partial f(g(\vec{x}))}{\partial x_i} = \frac{\partial f(\vec{u})}{\partial x_i} = \sum_{l=1}^{m} \frac{\partial f(\vec{u})}{\partial u_l} \cdot \frac{\partial u_l}{\partial x_i}$$

Towards backpropagation (1)

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Recall

$y_j^{(l+1)} = \sum_{i=0}^{M^{(l)}} w_{i,j}^{(l+1)} f_i^{(l)} + b_j^{(l+1)}$ and $f_j^{(l+1)} = h\left(y_j^{(l+1)}\right)$

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We need to compute $\frac{\partial E(x,y)}{\partial w_{i,j}^{(l)}}$ and $\frac{\partial E(x,y)}{\partial b_j^{(l)}}$

Recall

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Observation

E depends on all f_j^L, which depends on f_j^{L-1} ...

$$\frac{\partial E}{\partial w_{i,j}^l} = \frac{\partial E}{\partial f_j^l} \cdot \frac{\partial f_j^l}{\partial y_j^l} \cdot \frac{\partial y_j^l}{\partial w_{i,j}^l}$$

Towards backpropagation (1)

Goal

We need to compute $\frac{\partial E(x,y)}{\partial w_{i,j}^{(l)}}$ and $\frac{\partial E(x,y)}{\partial b_j^{(l)}}$

Recall

$y_j^{(l+1)} = \sum_{i=0}^{M^{(l)}} w_{i,j}^{(l+1)} f_i^{(l)} + b_j^{(l+1)}$ and $f_j^{(l+1)} = h\left(y_j^{(l+1)}\right)$

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Contains all derivatives from L to l

Backpropagation for $w_{i,j}^{l}$

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Backpropagation for b_j^l

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Backpropagation for activation h / loss E

Gradient step

$$
\begin{aligned}
w_{i,j}^{(l)} &= w_{i,j}^{(l)} - \alpha \cdot \delta_j^{(l)} f_i^{(l-1)} \\
b_j^{(l)} &= b_j^{(l)} - \alpha \cdot \delta_j^{(l)}
\end{aligned}
$$

Recursion

$$
\begin{aligned}
\delta_j^{(l-1)} &= \frac{\partial h(y_i^{(l-1)})}{\partial y_i^{(l-1)}} \sum_{k=1}^{M^{(l)}} \delta_k^{(l)} w_{j,k}^{(l)} \\
\delta_j^{(L)} &= \frac{\partial E(f_j^{(L)})}{\partial f_j^{(L)}} \cdot \frac{\partial h(y_j^{(L)})}{\partial y_j^{(L)}}
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Note Assume L layers in total

Backpropagation Different notation

Notation We used scalar notation so far

Fact Same results can be derived using matrix-vector notation

$\rightarrow$ Notation depends on your preferences and background

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For us We want to implement backprop. from scratch, thus scalar notation is closer to our implementation
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$$\delta^{(l-1)} = \left(W^{(l)}\right)^T \delta^{(l)} \odot \frac{\partial h(y^{(l-1)})}{\partial y^{(l-1)}}$$

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vectorial derivative!

Hadamard-product / Schur-product: piecewise multiplication

Backpropagation Some remarks

Observation Backpropagation is a recursive algorithm
Use Dynamic programming for implementation
→ Start with output layer and the go back

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Important note
SGD is a general optimization approach
Backpropagation is a general way to compute gradients in directed acyclic graphs

Remark 3 With Neural Networks we combine both

Backpropagation Some implementation ideas

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Thus Implement neural networks layer-wise

- Each layer has activation function
- Each layer has derivative of activation function
- Each layer has weight matrix (either for input or output)
- Each layer implements delta computation
- Output-layer implements delta computation with loss function
- Layers are either connected to each other and recursively call backprop. or some “control” function performs backprop.

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Thus Arbitrary network architectures can be realised without changing learning algorithm

MLP Some ideas about architectures

Question So what is a good architecture?

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Some general ideas

- **Non-linear activation** A network should contain at least one layer with non-linear activation function for better learning
- **Sparse activation** To prevent over-fitting, only a few neurons of the network should be active at the same time
- **Fast convergence** The loss function / activation function should allow a fast convergence in the first few epochs
- **Feature extraction** Combining multiple layers in deeper networks usually allows (higher) level feature extraction

Data mining From MLP to Deep Learning

Observation

- **1 perceptron** Separates space into two sets
- **Many perceptrons in 1 layer** Identifies convex sets
- **Many perceptrons in 2 layer** Identifies arbitrary sets

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$\rightarrow$ Given enough hidden units, a MLP is able to represent any "well-conditioned" function **perfectly**

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But That does not necessarily mean, that we will find it!

- Usually we cannot afford exponentially large networks
- Learning of $\vec{w}$ might fail due to data or numerical reasons

Deep Learning From MLP to Deep Learning

So... How did Deep Learning become so popular?

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Trade width for depth
$\rightarrow$ Extract features and combine them in later layers

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Zhang et. al 2017
$\mathcal{O}(N+d)$ weights are enough for sample of size N in d dimensions
$\rightarrow$ "One" neuron per sample

But This introduces new challenges

Deep Learning Vanishing gradients

Observation 1 $\sigma(z) = \frac{1}{1+e^{-\beta \cdot z}} \in [0, 1]$

Observation 2 $\frac{\partial \sigma(z)}{\partial z} = \sigma(z) \cdot (1 - \sigma(z)) \in [0, 0.25\beta]$

Observation 3 Errors are multiplied from the next layer

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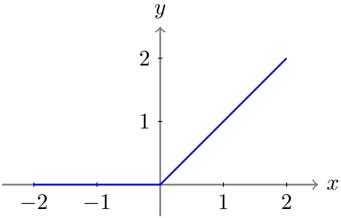
So far No fundamental solution found, but a few suggestions

- Change activation function
- Exploit different optimization methods
- Use more data / carefully adjust stepsizes
- Reduce number of parameters / depth of network

Deep Learning ReLu activation

Rectified Linear (ReLu)



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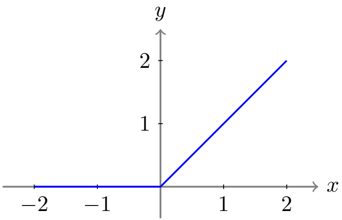

$$h(z) = \begin{cases} z & \text{if } z \geq 0 \\ 0 & \text{else} \end{cases} = max(0, z)$$

$$\frac{\partial h(z)}{\partial z} = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{else} \end{cases}$$

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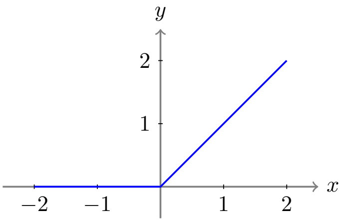

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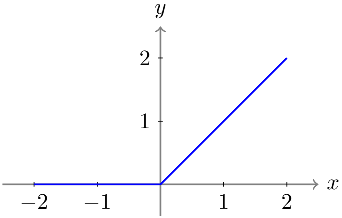

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But Usually that is not a problem

- **Practical** $z = 0$ is pretty rare, just use 0 there. It works well
- **Mathematical** There exists a subgradient of $h(z)$ at 0

Deep Learning ReLu activation (2)

Subgradients A gradient shows the direct of the steepest descent
$\Rightarrow$ If a function is not differentiable, it has no steepest descent
$\Rightarrow$ There might be multiple (equally) "steepest descents"

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For ReLu We can choose $\frac{\partial h(z)}{\partial z}\Big|_{z=0}$ from $[0, 1]$
Big Note Using a subgradient does not guarantee that our loss function decreases! We might change weights to the worse!

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Nice properties of ReLu

- Super-easy forward, backward and derivative computation
- Either activates or deactivates a neuron (sparsity)
- No vanishing gradients, since error is multiplied by 0 or 1
- Still gives network non-linear activation

Deep Learning Loss function

Usually Squared error

$$E = \frac{1}{2}\left(y - f^{(L)}\right)^2 \Rightarrow \frac{\partial E}{\partial f^{(L)}} = -\left(y - f^{(L)}\right)$$

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Thus

$$\delta_j^{(L)} = -\left(y - f^{(L)}\right) \cdot \frac{\partial h(y^{(L)}))}{\partial y^{(L)}} \rightarrow \text{ small for sigmoid!}$$

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Mohamed et. al 2009

Cross-entropy

$$E = -\left(y \ln\left(f^{(L)}\right) + (1-y)\ln\left(1 - f^{(L)}\right)\right) \Rightarrow \frac{\partial E}{\partial f^{(L)}} = \frac{f^{(L)} - y}{(1-f^{(L)})f^{(L)}}$$

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Mohamed et. al 2009

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Idea View y and $\widehat{y}$ as categorical distribution

Then Minimize distance between both distributions

Nice bonus

$\delta_j^{(L)} = \frac{f^{(L)}-y}{(1-f^{(L)})f^{(L)}} \cdot \frac{\partial h(y^{(L)}))}{\partial y^{(L)}} = f^{(L)} - y$ cancels small sigmoids

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Mohamed et. al 2009

Cross-entropy

$E = -\left(y \ln\left(f^{(L)}\right) + (1-y)\ln\left(1 - f^{(L)}\right)\right) \Rightarrow \frac{\partial E}{\partial f^{(L)}} = \frac{f^{(L)} - y}{(1-f^{(L)})f^{(L)}}$

Idea View y and $\widehat{y}$ as categorical distribution

Then Minimize distance between both distributions

Nice bonus

$\delta_j^{(L)} = \frac{f^{(L)} - y}{(1-f^{(L)})f^{(L)}} \cdot \frac{\partial h(y^{(L)}))}{\partial y^{(L)}} = f^{(L)} - y$ cancels small sigmoids

Important

Make sure that $\sum f^L = 1 \rightarrow$ This is called softmax layer

Data Mining Convergence of SGD

Recall We use the SGD framework

$\vec{\theta} = random()$
while ERROR **do**
  choose random $(\vec{x}, y) \in \mathcal{D}$
  $\vec{\theta} = \vec{\theta} - \alpha_t \cdot \frac{\partial E(x,y)}{\partial \vec{\theta}}$
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Bottou etal. 2017 SGD converges if

1) $\frac{\partial E(x,y)}{\partial \vec{\theta}} = \nabla_\theta \mathbb{E}[\nabla_\theta E(\mathcal{D})]$ is unbiased estimator of true gradient
2) $\alpha_t \rightarrow 0$, if E is not convex

Note If E is non-convex we may find a local minima

SGD Stepsize

What about the stepsize?

- If its to small, you will learn slow ($\rightarrow$ more data required)
- If its to big, you might miss the optimum ($\rightarrow$ bad results)

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But Stepsize is problem specific as always!

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Practical suggestion Simple heuristic

- Try out different stepsizes on small subsample of data
- Pick that one that most reduces the loss
- Use it for on the full dataset

Sidenote Changing the stepsize while training also possible

SGD Momentum

$$
\begin{aligned}
\Delta\widehat{\theta}^{old} &= \alpha_1 \cdot \nabla_\theta E(\mathcal{D}, \widehat{\theta}^{old}) + \alpha_2 \Delta\widehat{\theta}^{old} \\
\widehat{\theta}^{new} &= \widehat{\theta}^{old} - \Delta\widehat{\theta}^{old}
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Theoretically more sound

- **Nesterov 1983 / Sutskever et. al 2013** Nesterov momentum
- **Tielman et al. 2012 / Graves 2013** RMSProp
- **Kingma and Lei Ba 2015** Momentum tuned for SGD: ADAM

...and many more AdaGrad, AdaMax, AdaDelta, . . .

Bonus Methods often give heuristic for step-size

SGD Utilize parallelism

(Mini-)Batch

Compute derivatives on batch and average direction

$\rightarrow$ parallel computation $+$ only 1 parameter update

$$\widehat{\theta}^{new} = \widehat{\theta}^{old} - \alpha \cdot \frac{1}{K} \sum_{i=0}^{K} \nabla_\theta E(\vec{x}_i, \widehat{\theta}^{old})$$

Note That works particularly well on GPUs or FPGAs ...

SGD Initial solution

For SGD

Need initial solution θ

Common in practice

Bias $b = 0$, weights $w_{ij}^l \sim \mathcal{N}(0, 0.05)$

Bias $b = 0$, weights $w_{ij}^l \sim \mathcal{U}(-0.05, 0.05)$

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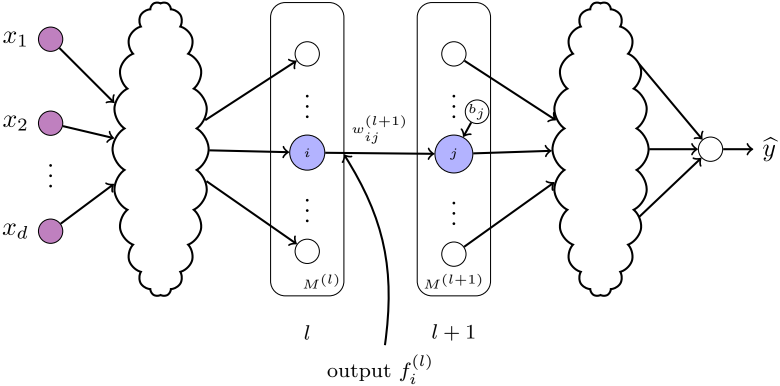
Why care?

$$\begin{aligned} \delta^{(L)} &= \frac{\partial E(f^{(L)})}{\partial f^{(L)}} \cdot \frac{\partial h(y^{(L)})}{\partial y^{(L)}} = -(y_i - f_j^L) f_j^L (1 - f_j^L) \\ \delta^{(L)} &= 0 \text{ if } f_j^L = 0 \text{ or } f_j^L = 1 \end{aligned}$$

Thus We stuck in local minima if we have a bad initialization

Deep Learning Slow learning rate



Recall

Input of neuron depends on output of previous neurons

Deep Learning Slow learning rate (2)

Observation During training, activations change over time
Thus Input distribution for neurons also change over time

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Ioffe and Szegedy 2015
Internal covariate shift of activations

Idea
Normalize neuron inputs to be zero mean / unit variance

Deep Learning Slow learning rate (3)

During training

Given mini batch $\mathcal{B} = \{(y_j^l)_i\}_{\{i=1,\ldots,K\}}$, compute

$$\begin{aligned}\overline{y}_j^l &= \frac{1}{K}\sum_{i=0}^{K}(y_j^l)_i \\ (y_j^l)_i &= \frac{(y_j^l)_i - \overline{y}_j^l}{\sqrt{\sigma_{\mathcal{B}} + \varepsilon}}\end{aligned}$$

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\end{aligned}
$$

Note

During inference there is usually no mini batch

Thus

Estimate y_j^l over all training data while training

Data Mining Large models tend to overfit

Common intuition 1

Large models tend to memorize data $\rightarrow$ no generalization

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Training error always decreases, but test error may increase again

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Limit SGD to volume around initial solution

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Common practice Early stopping

$\rightarrow$ Use fixed number of iterations or timesteps

Deep Learning Force redundancy

Hinton et al. 2013 / Srivastava et al. 2014 DropOut

Ignore neuron with probability p during forward-pass in training

$\rightarrow$ sometimes $f_j^l = 0$ during training

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Ignore neuron with probability p during forward-pass in training
$\rightarrow$ sometimes $f_j^l = 0$ during training

Wan et al. 2014: DropConnect
Ignore weight with probability p during forward-pass in training
$\rightarrow$ sometimes $w_{i,j}^l = 0$ during training

Summary

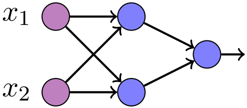
Important concepts

- **For parameter optimization** we define a loss function
- **For parameter optimization** we use gradient descent
- **Neurons** have activation functions to ensure non-linearity and differentiability
- **Backpropagation** is an algorithm to compute the gradient
- **Deep Learning** requires new activation functions
- **Deep Learning** requires new loss functions
- **Deep Learning** sometimes require a lot fine-tuning

Homework

Homework until next meeting

- Implement the following network to solve the XOR problem



- Implement backpropagation for this network
  - Try a simple solution first: Hardcode one activation / one loss function with fixed access to data structures
- If you feel comfortable, add new activation / loss functions

Tip 1: Verify that the proposed network uses 9 parameters

Tip 2: Start with $\alpha = 1.0$ and 10000 training examples

Note: We will later use `C`, so please use `C` or a `C`-like language

Question: How many iterations do you need until convergence?